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HYPOTHESIS ON UNIVERSAL EJECTION PROPERTIES
OF TURBULENT JETS OF GAS AND ITS APPLICATION

By

O. V. Yakovlevskii

In the instance of a turbulent flow of liquids the transposition of Navier-Stokes equations by replacing each of the parameters of the moving medium with a sum of a certain average value and a pulsating component leads to a very complex system, the solution of which is usually attempted by two methods; namely, by statistical means or by the use of certain semiempirical relationships. Certain results are obtained through the semiempirical theory of turbulence, partially solving the problem of the propagation of a free turbulent current of an incompressible liquid in a motionless medium. However, due to the absence of corresponding semiempirical connections essential for closure of the equation system describing the propagation of a gas jet in a moving medium, this problem has remained unsolved up to this time. Only after research done by G. N. Abramovich /1/, who, developing the ideas of L. Prandtl, found a semiempirical relationship which associates the intensity of a turbulent expansion of the jet with the values of the flow rate on the boundaries of the agitation region, was it possible to solve the problem of the propagation of the jet of liquid within the flow of the same physical properties, and the problem of a gas jet in a moving and motionless medium /2 - 4/.

Comparison of the solutions obtained by the experimental data of various authors /3/ indicates that a semiempirical theory in general terms correctly reflects the actual propogation process of the gas jet. A systematic discrepancy in the calculation and test results observed at $m_b (= u_\infty/u_o) > 0.35$, is qualitatively explained by the influence of a factor of initial turbulence of agitating currents which was not taken into account in the theory. However, the calculations indicate that the initial stage of the turbulence must be very high — to the order of 10 — 15%. Since during a set flow in the various channels the stage of turbulence does not ordinarily exceed 1 — 5% (i. e. these values determine the initial turbulence of the jet and the accompanying flow), it is evident, that the indicated qualitative explanation lacks an essential experimental substantiation.

A calculation of the associated discharge in jets of gas, propagating in a still or a moving medium, indicate that, in the area of accord of the experimental and calculated characteristics of a jet, the same ejection properties as those of the corresponding (with the same initial impulse and area of an exhaust jet cross-section) submerged jet of an incompressible liquid are retained.

This fact has furthered the idea regarding the universality of the ejection properties of a turbulent jet, regardless of the external conditions of its development. The specified hypothesis allows, without any specific difficulties, the use of a well-known and extensively proven theory of a submerged turbulent jet of an incompressible liquid for finding laws of jet propagation under the most difficult conditions (anisothermal, wasteful discharge, dynamic compressibility, and presence of a longitudinal pressure gradient, etc.).

We note that the indicated hypothesis is based on the following considerations: since the basic characteristic of a jet is its impulse, and the main property is the ejection of the substance from the surrounding medium, it is natural to assume that, during maintenance of a constant impulse of a jet, its ejection properties remain invariable.

The hypothesis on universal ejection properties of turbulent jets is analytically formulated in the following manner. If a given jet (index i), propagating under arbitrary conditions, and a standard, submerged, isothermic, free jet (index a) both discharge from the same nozzle and in the initial cross section have identical impulses,

$$M_i = M_a, \tag{0.1}$$

then these jets also possess similar ejection properties. In other words, the laws of the growth in the mass of the jet concur due to involvement in it of a substance from the surrounding medium and fulfillment of condition (0.1) for a given and a standard jet:

$$\frac{dG_i}{dx} = \frac{dG_a}{dx}. \tag{0.2}$$

Here G is the gas discharge in the arbitrary cross section of the jet, and x is the longitudinal coordinate read from the section of the nozzle.

It is easily seen that by satisfaction of condition (0.1) and by equality of the stages of the outlet apertures of the nozzles (having the same geometrical form), the relationship of discharge of a standard and an arbitrary jet is expressed by formula

$$\frac{G_{a0}}{G_{i0}} = \sqrt{\frac{\rho_{a0}}{\rho_{i0}}} = \sqrt{\rho^{\circ}} \qquad (0.3)$$

where, hypothetically, ρ_{ao} is the density of the surrounding medium into which the jet discharges.

Relationship (0.2), considering formula (0.3), is reduced to a dimensionless form

$$\frac{dG_i^{\circ}}{dx^{\circ}} = \sqrt{\rho^{\circ}}\,\frac{dG_a^{\circ}}{dx^{\circ}} \qquad \left(G^{\circ} = \frac{G}{G_0},\ x^{\circ} = \frac{x}{r_0}\right) \qquad (0.4)$$

where r_0 is the initial radius of an axisymmetrical jet or the initial half-width of a plane parallel jet.

The hypothesis proposed above may also be formulated in the following manner: the intensity of the growth in the mass of the jet (its ejection capability) is proportional to its initial impulse dG/M = idem.

If there are two jets discharging from geometrically identical nozzles, then it is easily seen that the relationship of the given increments of discharge ($dG^{\circ} = dG/G_0$) in these jets may be expressed by formula

$$\frac{dG_1^{\circ}}{dG_2^{\circ}} = \sqrt{\frac{M_1}{M_2}\,\frac{F_2}{F_1}\,\frac{\rho_2}{\rho_1}}\,.$$

With the same initial impulses and stages of the outlet apertures we have $dG_1^{\circ} = \sqrt{\rho^{\circ}}\,dG_2^{\circ}$, which is analogous to the above obtained relationship (0.4).

Finally, it may be seen that the proposed hypothesis in the case of a submerged gas jet corresponds in the first approximation to the assumption on the universality of the profile of the relative transverse velocity component: v/u_m = idem.

This assumption, in conjunction with determinations of many investigators /3/ establishes the fact that the profile similarity of a longitudinal velocity component (v/u_m = idem) is equivalent to the hypothesis on the universality of a kinematic flow pattern in gas jets.

We show this by using an axisymmetrical jet as an example where the increase of the rate of flow in an arbitrary section of the submerged jet $dG = 2\pi g \rho_\infty v_\infty r dx$ is represented in a dimensionless form:

$$\frac{dG^\circ}{dx^\circ} = 2\frac{r}{r_0}\frac{u_m}{u_0}\rho^\circ\frac{v_\infty}{u_m} \qquad \left(\rho^\circ = \frac{\rho_\infty}{\rho_0}\right).$$

As will be shown below (2.4), the following relationship is valid in the examined instance:

$$\frac{r}{r_0}\frac{u_m}{u_0} = \frac{1}{\sqrt{\rho^\circ\Phi(u_m/u_0, \rho^\circ)}}$$

where for not very high values ρ°, function Φ may be replaced by constant $\Phi(u_m/u_0, \rho^\circ) \approx$ const. Then

$$\frac{dG^\circ}{dx^\circ}\frac{1}{\sqrt{\rho^\circ}} \approx \text{const}\frac{v_\infty}{u}.$$

An approximate accord between the expressed hypothesis (0.4) and the assumption on the kinematic universality of jets v/u_m = idem, u/u_m = idem, follows from this.

Analysis of known experimental and calculated data of various authors shows that the growth in mass along the submerged jet of an incompressible liquid is satisfactorily described by the following generalized relationships:

for the basic section of an axisymmetrical jet

$$G^{o}\ (x^{o}) = \alpha_1(x^{o} + \beta_1) \qquad (0.5)$$

for the basic section of a plane-parallel jet

$$G^{o}\ (x^{o}) = \alpha_2 \sqrt{x^{o} + \beta_2} \qquad (0.6)$$

for the instance of an initial section of a plane jet

$$G^{o}\ (x^{o}) = \gamma x^{o} + 1 \qquad (0.7)$$

In the first approximation this formula may also be used during calculation of the initial section of an axisymmetrical jet.

With regard to the formulas which determine the law of ejection of a turbulent jet for a substance from the surrounding medium, it should be noted that they noticeably differ with the various authors, although being analogous in their structure, and in general satisfying relationships (0.5) - (0.7). For instance, for the main section of an axisymmetrical jet of an incompressible liquid, coefficient α_1, according to the data of several authors, deviates from 0.11 to 0.20; actually, however, the most frequently encountered values α_1 according to data of articles /3 - 9/, lie in the interval 0.13 - 0.16. The basic cause of similar divergencies, as well as discrepancies of the formulas for determining attenuation of the axial velocity of the jet, is due, in our opinion, to the illegibility in processing experimental data. The authors seldom cite data on the distribution of velocity and other parameters of flow in the initial section of the jet; and on the initial turbulence, they refer the axial velocity usually to either a maximum or to an average (by discharge or by impulse) velocity value in the initial section without specifying which of the indicated means is used. As will be seen below, calculation of the irregularity of the flow during its outlet from the nozzle (characterized by coefficients n_{1u}, n_{2u}, n_i) introduces essential corrections into the calculation and should be performed during the process of the test data when the calculated and test results are compared.

Shown here are coefficient values in formulas (0.5) - (0.7) corresponding to the available test data:

$\alpha_1 = 0.155, \quad \beta_1 = -4.3, \quad \alpha_2 = 0.376, \quad \beta_2 = 0, \quad \gamma = 0.0362.$

We note that, due to an accumulation of experimental data and also after a careful process of the already known test data on isometric submerged jets during fixed conditions in the initial section, these values may possibly have to be changed somewhat (in particular, the coefficients in formulas (0.5) - (0.7) may become dependent upon the initial stage of the turbulence in the jet). Thus, in order to preserve the generality of further computations, for the present we will use the laws of ejection in the form of generalized relationships (0.5) - (0.7).

It is not difficult to obtain the law of ejection for an arbitrary jet from formulas (0.4) - (0.7). Thus, for the initial section of a jet we have:

$$G^\circ = \sqrt{\rho^\circ}\,\gamma x^\circ + 1 \; ; \qquad (0.8)$$

and for the basic section of an axisymmetric jet we obtain

$$G^{\circ} = \sqrt{\rho^{\circ}}\alpha_1(x^{\circ} + \beta_1) \cdot \qquad (0.9)$$

By derivation of these dependencies, integration constants are determined from the condition of conformity of the relationships obtained with the law of ejection for an isothermic submerged jet at $\rho^{\circ} = 1$. Generally speaking, these integration constants may be the functions of the initial parameters of the jet (for example, ρ°); but we will consider them here as constants.

As may be noted, the values characterizing kinematics of the surrounding medium are not entered on the right sides of relationships (0.8) - (0.9). This indicates that, in accord with the accepted hypothesis, the relative velocity of the associated flow does not influence the value of the added mass of the jet. This fact is very challenging and may present interest in problems dealing with mixture and combustion in a flow. It particularly shows that the mixing of a jet with the surrounding medium which develops a growth in the mass of the gas drawn into the jet will also occur at equal velocities of the jet and the associated flow. However, if the impulse of the associated flow is greater than that of the jet, then the flow will be determined by characteristics of the associated current; thus, in this instance the ejection capability (proportional to the initial impulse in accordance with the accepted hypothesis), of the associated flow and not of the jet, plays a predominant role.

The above hypothesis expressed by formula (0.4) is an alternate for an equation of propagation of a turbulent gas jet, proposed by the author and G. N. Abramovich in monograph /3/; and this equation, together with equations of impulse conservation and excess heat content, forms a closed system for calculating gas jets in a moving medium /3/.

In separate examples below, results obtained by practical application of the proposed hypothesis, as well as their comparison with corresponding experimental data, will be shown.

1. Initial Region of the Liquid Current in the Associated Flow.

Let us examine the initial region of the isothermic turbulent jet propagated in a moving medium. Based on the hypothesis concerning the universality of the profile of an infinite excessive velocity, as shown in /1/

$$\Delta u^{\circ} = \frac{u_1 - u}{u_1 - u_2} = (1 - \eta^{\frac{3}{2}})^2 \qquad \left(\eta = \frac{y - y_2}{y_1 - y_2}, \quad y_1 - y_2 = b\right)$$

from the equation of momentum conservation, and the rate and cross-sectional equilibrium of the liquid for the contour Ky_1y_2OM (Fig. 1), the following dependence of the relative ordinate of the inner boundary of the jet boundary layer from parameter $m = u_2/u_1$ can be obtained:

$$\frac{y_1}{b} = 0.416 + 0.134\,m\,. \tag{1.1}$$

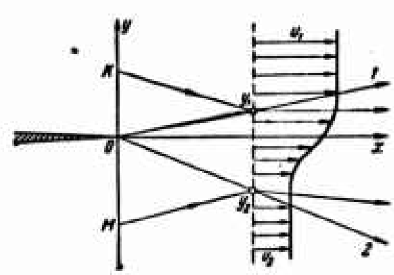

Figure 1. A pattern of the flow in the agitation zone of the initial area of the jet.

To solve the problem we now have to find the change in thickness of the free boundary layer b along the length of the jet. For this purpose G. N. Abramovich proposed /1/ a relationship derived on the basis of L. Prandtl's ideas on the mechanism of agitation in the jet. However, as mentioned above, this $m > 0.35$ gives results which do not correspond with reality. Let us attempt to use our hypothesis on the universality of the ejection properties of a turbulent jet instead of the mentioned relationship. For this purpose we calculate the quantity of liquid in the arbitrary cross-section of the initial region of a plane-parallel jet

$$G = 2g\rho_1 u_1 (b_0 - y_1) + 2g \int_{y_1}^{y_1} \rho u dy.$$

For the instance $\rho^\circ = 1$ here we obtain

$$G^\circ = \frac{G}{G_0} = 1 + \frac{b}{b_0}\left(0.55 + 0.45\, m - \frac{y_1}{b}\right). \qquad (1.2)$$

Here, by substituting dependence $C^\circ(x^\circ)$ according to formula (0.8) with $\rho^\circ = 1$ and replacing value y_1/b according to equation (1.1), we get

$$b^\circ = \frac{\gamma}{0.134 + 0.316\, m}\, x^\circ.$$

Here with $\gamma = 0.0362$ it follows

$$b^\circ = \frac{0.27}{1 + 2.36\, m}\, x^\circ, \qquad (1.3)$$

or

$$b^* = \frac{b^\circ}{(b^\circ)_{m=0}} = \frac{1}{1 + 2.36\, m}. \qquad (1.4)$$

In Figure 2 dependence (1.4) is compared with the corresponding test data of the author /2/ and B. A. Zhestkov /4/, and also with the analogous dependence obtained by G. N. Abramovich /1/;

$$b^{*} = \frac{1 - m}{1 + m} \cdot \qquad (1.5)$$

It follows from a comparison of the calculated and experimental results that formula (1.4), based on the adopted hypothesis in a diapason of $0.35 < m < 1$, corresponds better with the test points than relationship (1.5); but at $m < 0.35$ both calculated curves satisfactorily correspond with experimental values.

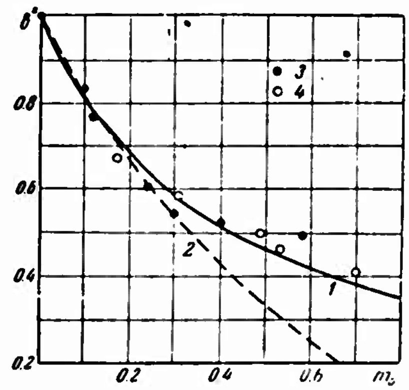

Figure 2. Dependence of the reduced coefficient in the thickening of the agitation area of an isothermic jet ($\rho^{\circ} = 1$) from parameter m; curve 1 by formula (1.4); curve 2 by formula (1.5); points 3 by tests of O. V. Yakovlevskii /2/, and 4 by tests of B. A. Zhestkov /4/.

2. Turbulent Jet of Heated Gas (with Variable Thermodynamic Properties) in an Associated Flow. Let us examine the basic region of a turbulent jet of heated or cooled gas flowing into a motionless or a moving medium of the same composition as the substance of the jet (Fig. 3). Lately, in connection with the development of various

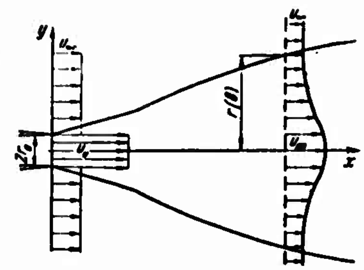

Figure 3. Schematic drawing of the flow in the initial area of the jet propagating in an associated flow.

types of systems, a considerable interest has arisen toward jets of highly heated gas (to 10 000 - 20 000°K).

At these temperatures, a physico-chemical conversion (dissociation, ionization), is observed in gases, causing a substantial change in its thermo-dynamic properties, particularly in heat-capacity, which, under these circumstances, cannot be considered as constant as is usually accepted during calculations of anisothermal jets /3/. A processing of the calculation data on the thermo-dynamic properties of various gases, such as air /10/, shows that in the diapason of pressure P, variation from 0.001 to 1000 ama and temperature T from 1000 to 12 000°K (this limit evidently may be increased to 20 000°K), may be written to appear as the following equation of state

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}\left(\frac{i_0}{i}\right)^{\alpha} \qquad (2.1)$$

where i is the enthalpy, and exponent α for air has a value of 0.83. By preliminary calculations for other gases the exponent has the same order; thus, in the case of hydrogen $\alpha = 0.92$. Further, we will use the equation of state, appearing in the form of (2.1) with $P = P_0$, assuming that the free turbulent jets are isobaric.

Test data on jets of heated air /3/ allow us to consider that their profiles of relative excessive velocity, like in isothermal jets, are universal and may be satisfactorily expressed by the following dependence:

$$\Delta u^{\circ} = \frac{\Delta u}{\Delta u_m} = \frac{u - u_{\infty}}{u_m - u_{\infty}} = (1 - \xi^{\frac{3}{2}})^2 \quad \left(\xi = \frac{y}{r}\right). \tag{2.2}$$

Besides, let us assume that the profiles of the relative excessive enthalpy are also universal at gas temperatures not exceeding 1000°K, and due to constant, specific heat correspond with the profiles of the relative excessive temperature of the jet, which, as known /3/, are universal. Let us consider that the profiles of enthalpy are expressed by the dependence

$$\Delta i^{\circ} = \frac{\Delta i}{\Delta i_m} = \frac{i - i_{\infty}}{i_m - i_{\infty}} = 1 - \xi^{\frac{3}{2}}. \tag{2.3}$$

For determination of kinematic and thermodynamic parameters in the arbitrary point of the jet of heated gas, considering relationships (2.2) and (2.3), the change of axial velocity and enthalpy as well as the thickness of the jet remains to be found, depending on the distance to its initial section. For this purpose we arrange a system of 3 equations.

The first two equations express the laws of conservation of the amount of the momentum and the excessive heat content in the jet; and after simple transformations (e. g. /3/) they may be presented in the following form:

$$\Delta u_m^{\circ} [m_0 A_1 + (1 - m_0) A_2 \Delta u_m^{\circ}] = \frac{F_0}{F} \frac{n_{2u} - m_0 n_{1u}}{1 - m_0} \tag{2.4}$$

$$\Delta i_m^\circ [m_0 B_1 + (1 - m_0) B_2 \Delta u_m^\circ] = \frac{F_0}{F} \frac{p_0 n_i - n_{1u}}{p_0 - 1} \quad . \tag{2.5}$$

Here

$$\begin{aligned} A_1 &= \int_0^1 \frac{\rho}{\rho_{om}} \Delta u^\circ \frac{dF}{F}, & A_2 &= \int_0^1 \frac{\rho}{\rho_{om}} (\Delta u^\circ)^2 \frac{dF}{F} \\ B_1 &= \int_0^1 \frac{\rho}{\rho_{om}} \Delta i^\circ \frac{dF}{F}, & B_2 &= \int_0^1 \frac{\rho}{\rho_{om}} \Delta i^\circ \Delta u^\circ \frac{dF}{F} \\ n_{1u} &= \int_0^1 \frac{\rho_0}{\rho_{om}} \frac{u_0}{u_{om}} \frac{dF_0}{F_0}, & n_{2u} &= \int_0^1 \frac{\rho}{\rho_{om}} \left(\frac{u_0}{u_{om}}\right)^2 \frac{dF_0}{F_0} \\ n_i &= \int_0^1 \frac{\rho_0}{\rho_{om}} \cdot \frac{u_0}{u_{om}} \frac{i_0}{i_{om}} \frac{dF_0}{F_0}, & m_0 &= \frac{u_\infty}{u_{om}}, \quad p_0 = \frac{i_{om}}{i_\infty} \end{aligned} \tag{2.6}$$

index 0 corresponds to the initial section of the jet, ∞ to the surrounding medium, and index m to the axis of the jet.

Let us indicate that the above relationships are of a general character, and that during their derivation only the hypothesis on the universality of profiles Δu° and Δi° is used. Besides, these relationships are adaptable to a plane-parallel as well as an axisymmetric jet, in which case it becomes appropriate for these instances:

$$\frac{dF}{F} = \frac{dy}{b} = d\xi, \qquad \frac{dF}{F} = 2\frac{y}{r}\, d\left(\frac{y}{r}\right) = 2\xi d\xi \ .$$

The above introduced parameters n_{1u}, n_{2u} and n_i characterize non-uniformity of the velocity and the enthalpy (or the temperature) profiles in the initial section of the jet; in a jet with an absolute uniform velocity and enthalpy the parameters have identical values equal one.

It should be noted that during calculations of turbulent jets the distinction between the parameters of non-uniformity and one is often disregarded, thus producing results considerably different from actuality. This especially concerns calculation of the jet in the associated flow because, as seen by analysis of formula (2.4), the influence of coefficients n_{1u} and n_{2u} (by their distinction from one) rapidly increases with the increase of parameter m_o from its zero value.

To illustrate this, it is seen that at an established turbulent velocity profile in the initial section of the jet (i. e., when the thickness of the boundary layer is equal to the radius of the exit aperture) parameters $n_{1u} = 0.815$, $n_{2u} = 0.680$ satisfy the degree of dependence with exponent $^1/_7$.

We use dependence (0.9) as the third relationship for closing the calculation system, i. e., from now on, we will only examine axisymmetric jets, which are more frequently encountered in practice. The calculation method for a plane-parallel jet is analogous and based on relationships (2.4), (2.5) and the ejection law for a plane-parallel jet described by formulas (0.4) and (0.6).

It is easily shown that with the above introduced assumptions on the universality of profiles Δu^o and Δi^o, the relative gas discharge in an arbitrary section of the jet is determined by relationship

$$G^\circ = \frac{G}{G_0} = \frac{1}{n_{1u}} \frac{F}{F_0} [m_0 A_0 + (1 - m_0) A_1 \Delta u_m^\circ] \quad . \tag{2.7}$$

Here

$$G = \int_0^F \rho u dF, \qquad G_0 = \int_0^{F_0} \rho_0 u_0 dF_0, \qquad A_0 = \int_0^1 \frac{\rho}{\rho_{0m}} \frac{dF}{F} \quad . \tag{2.8}$$

From a common solution of equations (2.4), (2.5) and (2.7) we easily obtain interesting dependent variables Δu_m^o, Δi_m^o and F/F_0 as functions of the relative discharge G^o, the change of which is described by relationship (0.9) in dependence upon the dimensionless distance x°. We have:

$$\Delta u_m^\circ \frac{m_0 A_1 + (1-m_0) A_2 \Delta u_m^\circ}{m_0 A_0 + (1-m_0) A_1 \Delta u_m^\circ} = \frac{1}{G^\circ} \frac{n_{2u} - m_0 n_{1u}}{n_{1u}(1-m_0)}, \tag{2.9}$$

$$\Delta i_m^\circ = k_i \Delta u_m^\circ, \tag{2.10}$$

$$k_i = \frac{p_0 n_i - n_{1u}}{p_0 - 1} \frac{1-m_0}{n_{2u} - m_0 n_{1u}} \frac{m_0 A_1 + (1-m_0) A_2 \Delta u_m^\circ}{m_0 B_1 + (1-m_0) B_2 \Delta u_m^\circ}. \tag{2.11}$$

It will become evident from the following that value k_i slightly changes in respect to the length of the jet with uniform profiles of velocity and enthalpy in the initial section ($n_{1u} = n_{2u} = n_i = 1$), and that this value is close to the value for an isothermic submerged jet $k_i = 0.753$ (in the case of a plane-parallel isothermic jet $k_i = 0.860$). In this manner relationships (2.10) and (2.11) almost correspond to the linear relationship between the excessive axial enthalpy and the velocity of the jet.

For determining dependencies of $\Delta u_m^\circ(x^\circ)$ and $\Delta i_m^\circ(x^\circ)$, it is necessary to know the values of A_i and B_i. Since profiles Δu° and Δi° are in the cross-sections, like functions ξ, they are known; but value ρ/ρ_{om} with the use of the equation of state (2.1) may be presented in the form

$$\frac{\rho}{\rho_{om}} = \frac{p_0^\alpha}{[1 + (p_0 - 1)\Delta i^\circ \Delta i_m^\circ]^\alpha}; \tag{2.12}$$

then, following integration, the dependence of coefficients A_i and B_i on values p_0 and Δi_m° may be found. However, for fractional values α indicated above, the integrals in formulas (2.6) during substitution of dependencies (2.12) into them are not used in the elementary functions; and thus, for their determination numerical calculations are performed in the diapason for the change in the value $(p_o - 1)\,\Delta i_m^\circ$ from 0.75 to 20.0 at $0.5 \leq \alpha < 1$. Empirical analysis of the results of these calculations has shown that, within the indicated limits, values A_i and B_i can be calculated with sufficient accuracy for practical use by the following formulas:

$$A_0 = \frac{p_0^\alpha}{[1+0.280\,(p_0-1)\,\Delta i_m^\circ]^\alpha}, \qquad A_1 = \frac{0.257\,p_0^\alpha}{[1+0.650\,(p_0-1)\,\Delta i_m^\circ]^\alpha},$$
$$A_2 = \frac{0.134\,p_0^\alpha}{[1+0.760\,(p_0-1)\,\Delta i_m^\circ]^\alpha}, \tag{2.13}$$
$$B_1 = \frac{0.428\,p_0^\alpha}{[1+0.500\,(p_0-1)\,\Delta i_m^\circ]^\alpha}, \qquad B_2 = \frac{0.178\,p_0^\alpha}{[1+0.730\,(p_0-1)\,\Delta i_m^\circ]^\alpha}.$$

Substitution of relationships (2.13) with calculation of formula (2.10) in equation (2.9) and replacement of value G° in formula (0.9) during the use of apparent equality

$$\rho^\circ = p_o^{\alpha} \tag{2.14}$$

permits us to obtain the relation between the axial excessive velocity and the distance from the nozzle, in a final form:

$$\frac{\Delta u_m^\circ}{\psi_{12}}\,\frac{1.92\,\psi_{12}\mu_0 + \Delta u_m^\circ}{3.895\,\psi_{01}\mu_0 + \Delta u_m^\circ} = \frac{1.92\,\nu_u}{p_0^{\frac{\alpha}{2}}\,a_1\,(x^\circ + \beta_1)}. \tag{2.15}$$

Here

$$\psi_{01} = \left[\frac{1+0.650\,(p_0-1)\,k_i\Delta u_m^\circ}{1+0.280\,(p_0-1)\,k_i\Delta u_m^\circ}\right]^\alpha, \qquad \psi_{12} = \left[\frac{1+0.760\,(p_0-1)\,k_i\Delta u_m^\circ}{1+0.650\,(p_0-1)\,k_i\Delta u_m^\circ}\right]^\alpha,$$
$$\mu_0 = \frac{m_0}{1-m_0}, \qquad \nu_u = \frac{n_{2u} - m_0 n_{1u}}{n_{1u}\,(1-m_0)}. \tag{2.16}$$

The change in enthalpy Δi_m° along the axis of the jet is determined by means of relationship (2.10). Coefficient k_i included in the calculated equations may be determined in the first approximation by formula (2.11) in which values A_1, A_2, B_1, B_2 are substituted, having been calculated from relationships (2.13) with the use of the coupling approximation

$$\Delta i_m{}^\circ = 0.753 \frac{p_0 n_i - n_{1u}}{p_0 - 1} \frac{1 - m_0}{n_{2u} - m_0 n_{1u}} \Delta u_m{}^\circ \quad . \qquad (2.17)$$

The calculations show that determinations of value k_i determined in this manner are very close to values k_i found in the second approximation.

The axial temperature of the jet T_m during a known value $\Delta i_m{}^\circ$ is determined by tables of thermodynamic functions for a given gas (in the case of air – /10/) as a dependence $T_m = T_m (i_m)$, where

$$i_m = (i_0 - i_\infty) \Delta i_m{}^\circ + i_\infty \quad . \qquad (2.18)$$

The thickness of the jet ($r^\circ = r/r_0 = \sqrt{F/F_0}$) in the arbitrary section is found in equation (2.4) with the use of dependence $\Delta u_m{}^\circ (x^\circ)$ determined by relationship (2.15).

Thus, the above obtained equations allow a determination along the parameters of the jet and the associated flow in the initial section (m_0, p_0, n_{1u}, n_{2u}, n_i) of the change in the axial velocity and temperature, and also in the thickness of the jet, depending on the dimensionless coordinate x°. The value of the velocity and the enthalpy in the arbitrary point of the jet is determined through their known values on the axis by means of formulas (2.2) and (2.3).

In order to estimate the influence of parameter p_0 characterizing the difference in the thermo-dynamic properties of the jet and of the surrounding medium, on propagation of the jet let us examine the simplest case of a heated gas discharge into a still medium ($m_0 = 0$).

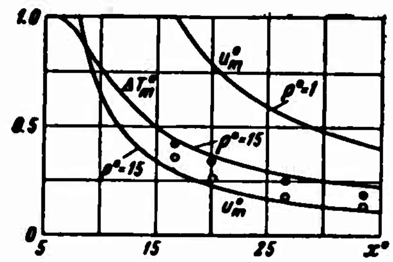

Figure 4. Change in axial velocity u°_m along the isothermic ($\rho^\circ = 1$) and the heated ($\rho^\circ = 15$) air jets; the curves by calculation; light points by tests of V. Ya. Bezmenov, and V. S. Borisov, at $\rho^\circ = 15$; the change in the axial temperature ΔT°_m along the axis of heated ($\rho^\circ = 15$) air jet; the curve by calculation; the points by tests with $\rho^\circ = 15$.

In this case, from equation (2.15) with $\mu_0 = 0$ after substitution of coefficients α_1 and β_1 by their corresponding numerical values, we obtain the following formula for axial velocity of the jet

$$u_m^\circ \left[\frac{1+0.650(\rho_0-1)k_t u_m^\circ}{1+0.760(\rho_0-1)k_t u_m^\circ}\right]^a = \frac{12.4\,v_u}{(x^\circ-4.3)\sqrt{\rho_0^a}} \qquad (2.19)$$

where

$$k_t = 0.753\,\frac{\rho_0 n_t - n_{1u}}{n_{2u}(\rho_0-1)}\left[\frac{1+0.730(\rho_0-1)k_t u^\circ_m}{1+0.760(\rho_0-1)k_t u^\circ_m}\right]^a \qquad (2.20)$$

or approximately,

$$k_t \approx 0.753\,\frac{\rho_0 n_t - n_{1u}}{n_{2u}(\rho_0-1)}\,. \qquad (2.21)$$

It follows from analysis of the law of attenuation of the axial velocity of an anisothermal jet (2.19) heated in relation to the surrounding medium that the axial velocity decreases at a considerably faster rate than that of an isothermic jet, during which value u°_m at a given value x° is inversely proportional to the square root of the density ratio of the surrounding medium and the jet. Figure 4 illustrates calculated dependencies u°_{m} (x°) for isothermic (p_{o} = 1, or ρ° = 1) or highly heated (p_{o} = 26, or ρ° = 15) jets of air discharging into a motionless medium.

Experimental points characterizing the axial velocity in a highly heated air jet, based on data of V. Ya. Bezmenov and V. S. Borisov*, are also given. As indicated by the results in Figure 4, the calculating formula (2.19) obtained on the basis of the hypothesis on the universality of ejection properties of turbulent jets satisfactorily corresponds with test data up to very high values of parameter p_{o}.

The attenuation curve of the excess axial temperature ΔT°_{m} in an isothermal jet is also shown in Figure 4; and the corresponding experimental values, which are evidently close to the values calculated, are also presented. It appears to us that this fact verifies the negligible effect of radiant heat exchange, which was not considered in these calculations, on the thermodynamic properties of highly heated turbulent jets.

In Figure 5 the calculated and experimental values of the thickness of a jet of highly heated air are compared, and a curve characterizing the change in the thickness of an isothermic jet along its length is represented. It is evident that the calculated thickness of a heated jet, which favorably corresponds with the experimental data, markedly exceeds the thickness of an isothermic jet at equal values x°.

The calculation and experimental data, given in Figure 5, permits

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* Data from this unpublished reference was made available to the author when composing this article.

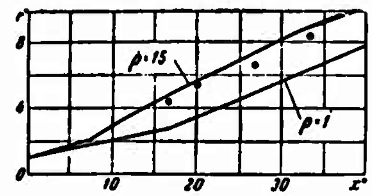

Figure 5. Thickening of jets of isothermic ($\rho^o = 1$) and heated ($\rho^o = 15$) air along axis x; curves by calculation; points, by tests with $\rho^o = 15$

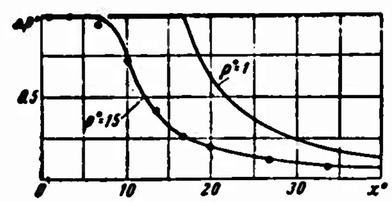

Figure 6. Change in the velocity head $\Delta p^o = \rho_m u_m^2/\rho_o u_o^2$ along the axis of jets of heated ($\rho^o = 15$) and isothermic ($\rho^o = 1$) air; curves, by calculation; points, by tests with $\rho^o = 15$

us to draw a conclusion that, in spite of the difference in the thicknesses of isothermic and heated submerged jets, the angle formed by the boundary of the basic region with axis x is evidently not dependent on parameter ρ^o, and its tangent has a constant value equal to 0.22.

Finally, a calculated curve characterizing the attenuation of the velocity head along the axis of a heated ($\rho^o = 15$) air jet is presented in Figure 6, and corresponding experimental points obtained by V. Ya. Bezmenov and V. S. Borisov are also plotted. A calculated curve of the axial velocity head in an isothermic air jet is also shown in this figure for the purpose of comparison. As may be seen, concurrence of the calculated and the test results in this instance ($\rho^o = 15$) are completely satisfactory.

Comparison of the velocity head data on the axis of the jet at

$\rho^{\circ} = 1$ and $\rho^{\circ} = 15$ is not beneficial to L. A. Vulie's /11/ hypothesis on the universality of velocity head distribution in gas jets.

The calculated dependence (2.19) well satisfies the test data also during small values ρ°. However, it should be noted that during $\rho^{\circ} \leqslant 3$ (i. e. at a temperature of a jet not exceeding 1000°K) the equation of state in an ordinary form should be used

$$\frac{\rho}{\rho_0} = \frac{p}{p_0}\,\frac{T_0}{T},$$

and the specific heat capacity c_p should be considered as constant. Then in all the above obtained relationships, $\alpha = 1$ and subsequently $p_0 = \rho^{\circ}$ should be considered. The calculations performed in this manner for the case of a heated ($\rho^{\circ} = 2$) submerged ($m_0 = 0$) jet are compared in Figure 7 with the corresponding test data /7/. A complete accord of calculation with experimental results is observed.

Comparison of the attenuation curves of the excessive axial velocity of an anisothermal jet ($\rho^{\circ} = 2.04$) propagating in the associated flow ($0 < m_0 < 0.5$) with the test data /12/ obtained for analogous conditions is given in Figure 8 where $n_{2u}/n_{1u} = 0.95$ is taken for all values m_0. By analyzing this figure, it follows that the hypothesis on the universality of the ejection properties of turbulent jets is valid in an ordinary case for propagating anisothermal jets in a moving medium.

3. Jet of an Incompressible Liquid in an Associated Flow.

For a more precise definition of the role of parameter m_0 in the propagation process of a turbulent jet, let us examine an individual case of an isothermic ($\rho^{\circ} = 1$) jet discharging into a flow of liquid

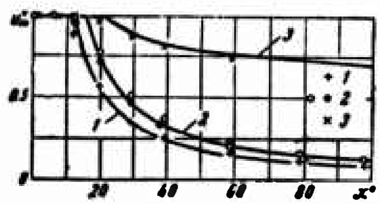

Figure 7. Comparison of calculated and experimental data on axial velocity u_{mo} of preheated ($\rho^\circ = 2$) and isothermic ($\rho^\circ = 1$) jets in the associated flow ($n_{1u} = 0.888$ and $n_{2n} = 0.806$); curves, by calculation; points, by tests of Yu. V. Ivanov and Ch. N. Sui /7/ for values: 1 for $\rho^\circ = 2$, $m_o = 0$; 2 for $\rho^\circ = 1$, $m_o = 0$, 3 for $\rho^\circ = 1$, $m_o = 0.65$

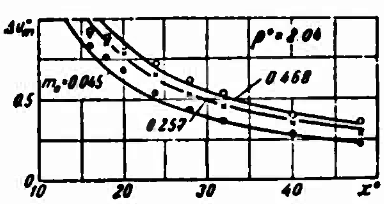

Figure 8. Change of the excess axial velocity Δu_m° in an isothermic ($\rho^\circ = 2.04$) jet propagating in the associated flow ($n_{2u}/n_{1u} = 0.95$); curves, by calculation; points, by tests of O. Pabst /12/

parallel to it. Under these conditions ($\rho^\circ = 1$), using formulas (2.2) and (2.6) we get

$$A_o = 1,\ A_1 = 0.257,\ A_2 = 0.134, \tag{3.1}$$

and the final equation determining the law of attenuation of axial velocity assumes the form

$$\Delta u^\circ_m \frac{1.92\,\mu_0 + \Delta u^\circ_m}{3.895\,\mu_0 + \Delta u^\circ_m} = \frac{12.4\,\nu_u}{x^\circ - 4.3}\ . \tag{3.2}$$

The thickness of the jet ($r^\circ = r/r_o = \sqrt{F/F_o}$) at a given value x° is determined from equation (2.4) with values of its entering coefficients determined by parity (3.1).

For illustrating the characteristics of propagation of an isothermic liquid jet in an associated flow (with $n_{1u} = n_{2u} = 1$), the respective curves of attenuation of the axial excess velocity Δu_m^o and the growth in the thickness of the jet r^o at $0 \leqslant m_o \leqslant 1$ are shown in Figures 9 and 10. As also follows from the theory of G. N. Abramovich /3/, the presence of the associated flow leads to an increase in the long range effect of the jet, but with the approach of parameter m_o to 1, the displacement of the axial velocity attenuation curve becomes weaker and weaker until it finally attains its end position with $m_o = 1$. With this, the thickness of the jet, as with $m_o \neq 1$, increases longitudinally; whereas, by theory /3/, the jet with $m_o = 1$ does not mix with the surrounding medium.

Comparison of the calculated values of the axial velocity of an isothermic jet in an associated flow ($0 < m_o < 1$) with the corresponding test data derived from reference /7/ is given in Figure 7. The initial irregularity of the jet corresponds with $n_{1u} = 0.888$ and $n_{2u} = 0.806$, and is taken into consideration with the additional information of Yu. V. Ivanov. It is evident that the hypothesis on the universality of the ejection properties of turbulent jets corresponds to actuality, also, in the case of propagation of an isothermic jet in the associated flow, the velocity of which does not exceed the initial velocity of the jet.

4. A Supersonic Jet During a Calculated and Uncalculated Regime of Discharge. We pause to examine a case of a submerged supersonic jet ($m_o = 0$). Propagation of a supersonic jet in the associated flow may be analyzed by analogous means. The equation of the conservation of momentum can be presented in the form /3/

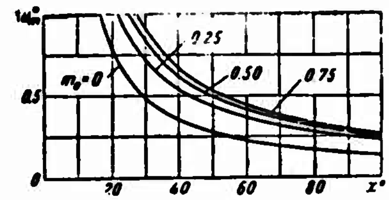

Figure 9. Influence of parameter m_0 on the attenuation in axial velocity of an incompressible liquid jet in the associated flow ($n_{1u} = n_{2u} = 1$)

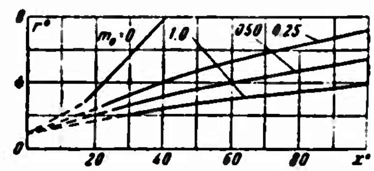

Figure 10. Influence of parameter m_0 on the thickness of an incompressible liquid jet in the associated flow ($n_{1u} = n_{2u} = 1$)

$$u_m^\circ A_2 = \frac{F_0}{F}\left[n_{2u}n + \frac{1-a\lambda_0^2}{a\lambda_0^2}(n-1)\frac{a}{1+a}\right]$$

$$\lambda_0 = \frac{u_0}{a_*}, \qquad a = \frac{k-1}{k+1}, \qquad n = \frac{P_0^*}{P^*} \approx \frac{P_0}{P_\infty}, \qquad k = \frac{c_p}{c_v}. \tag{4.1}$$

Here P_0^* and P^* are the actual and the calculated pressure in the receiver, λ_0 is the velocity coefficient, a_* is the critical speed of sound, and c_p and c_v are the specific heat. Value A_2 depends on the magnitude of $a\lambda_0^2$ and the axial velocity u_m°; this dependence is given in /3/.

The relative gas discharge in the arbitrary section of the jet is expressed by formula

$$G^\circ = \frac{F}{F_0}\,\frac{A_1}{n_{1u}{}^{n}}\,u_m{}^\circ\,. \qquad (4.2)$$

Here coefficient A_1 is a known /3/ of functions $a\lambda_o{}^2$ and $u_m{}^\circ$. By comparing relationships (4.1) and (4.2) we obtain

$$u_m{}^\circ = \frac{A_1}{A_2}\,\frac{1}{n_{1u}G^\circ}\left[n_{2u} + \frac{1-a\lambda_0{}^2}{a\lambda_0{}^2}\left(1-\frac{1}{n}\right)\frac{a}{1+a}\right]. \qquad (4.3)$$

If dependence $G^\circ = G^\circ\,(x^\circ,\,\rho^\circ)$ is substituted here by formulas (0.9), where

$$\rho^\circ = \frac{T_0{}^\circ}{T_\infty{}^\circ}\,\frac{1-a\lambda_0{}^2}{n}\,, \qquad (4.4)$$

and the following expression is introduced

$$\frac{A_1}{A_2} = 1.92\,S\,(u_m{}^\circ,\,a\lambda_0{}^2)\,, \qquad (4.5)$$

then formula (4.3) will appear as:

$$u_m{}^\circ = \frac{1}{n_{1u}}\left[n_{2u} + \frac{1-a\lambda_0{}^2}{a\lambda_0{}^2}\left(1-\frac{1}{n}\right)\frac{a}{1+a}\right]\frac{12.4\,S\,(u_m{}^\circ,\,a\lambda_0{}^2)}{\sqrt{\rho^\circ}\,(x^\circ-4.3)} \qquad (4.6)$$

In the first approximation function $S(u_m{}^\circ,\,a\lambda_o{}^2)$ with $0 \leqslant a\lambda_o{}^2 < 0.7$ and $u_m{}^\circ \leqslant 1$ may be considered as a constant value $S(u_m{}^\circ,\,a\lambda_o{}^2) \approx 1$, with accuracy acceptable for practical purposes.

The calculated curves of attenuation of the axial velocity in supersonic jets of air, with $a\lambda_o{}^2 = 0.311$ ($M_o = 1.5$) for several values of the parameter of incalculability n, are shown in Figure 11; and for an instance of a calculated discharge at supersonic speed with $a\lambda_o{}^2 = 0.644$ ($M_o = 3.0$) are shown in Figure 12. The corresponding experimental points derived from the works of B. A. Zhestkov and others /3/ are also presented here.

For the purpose of comparison, a curve of attenuation of the axial velocity for an isothermic jet of air with an initially low velocity

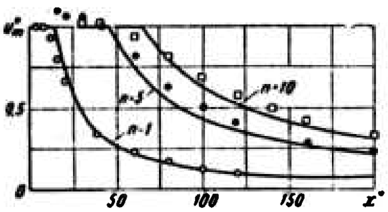

Figure 11. Calculation and experimental data on the change in the velocity on the axis of a supersonic (M_o = 1.5) air jet at calculable and incalculable regimes of discharge; curves, by calculation; points, by tests of B. A. Zhestkov and others /3/.

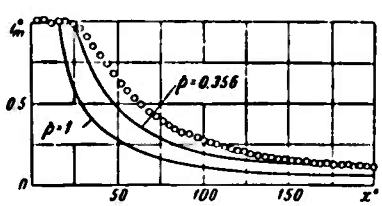

Figure 12. Calculation and experimental data on the change in the velocity in the axis of a supersonic ($M_o = 3.0$, $\rho^o = 0.356$) air jet; curves, by calculation; points, by tests of B. A. Zhestkov and others /3/.

($M_o \approx 0$, $\rho^o = 1$) is shown in Figure 12. It is evident that the supersonic jet has a considerably much longer range effect i. e., at a given value x^o its axial velocity u_m^o is higher than that of a jet with a low initial velocity. Analysis of these materials indicates that the calculations, performed on the basis of the hypothesis on the universality of the ejection properties of turbulent jets, also satisfactorily correspond with known test data for supersonic (including the incalculable) jets of air up to the discharge velocity of gas corresponding to Mach number $M_0 = 3$.

5. Liquid-Gas Jet. In conclusion, let us examine the discharge of a gas jet into a medium filled with liquid drops. The theory of this jet is evolved on the basis of the author's formula

$$\left(\frac{db}{dx}\right)_{m=0} \sim u_0 \int_0^b \rho dy \Big/ \int_0^b \rho u dy \qquad (5.1)$$

initially published in /3/.

Let us attempt to apply our hypothesis to this case, also. As shown in /3/, the equation of the conservation of momentum leads to the following relationship between the axial velocity and the cross-sectional area of a liquid gas jet.

$$u_m^\circ = N\frac{F_0}{F} \qquad (5.2)$$

where $N = 3.85$ for an axially symmetric jet. From the equation of the conservation of momentum of the initial amount of gas, the following relationship between the concentration of gas on the axis of the jet and the axial velocity, is obtained:

$$u_m^\circ = K\varkappa_{gm} \qquad (5.3)$$

where, in the case of an axially symmetric jet,

$$K = 1.45.$$

Further, it is easily seen that the discharge of a liquid gas mixture in an arbitrary section, may be expressed by formula

$$G^\circ = 0.428\frac{F}{F_0}\frac{u_m^\circ}{\varkappa_{gm}}. \qquad (5.4)$$

If instead of value G^0 we substitute here its dependence on x^0 by formula (0.9), then relationship (5.2) and (5.4) will form a closed system of equations which will allow a determination of the

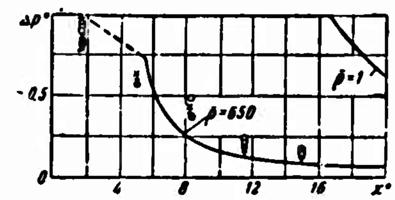

Figure 13. Comparison of calculated attenuation curves of the axial velocity head $\Delta p^{\circ} = \rho_m u_m^2/\rho_g u_g^2$ in a jet of air discharging into water ($\rho^{\circ} \approx 650$), and in a jet with a constant density ($\rho^{\circ} = 1$), with test data for a liquid gas jet /13/.

velocity and the concentration and thickness of the mixture on the axis of the jet.

In some cases the dynamic properties of a liquid gas jet determined by its velocity head are of interest. As shown in /3/, the velocity head of the mixture on the axis of the main area is related to the initial velocity head of the air jet and may be expressed by the following relationship of approximation:

$$\frac{\rho_m u_m^2}{\rho_g u_g^2} \approx K u_m^{\circ} . \tag{5.5}$$

By substituting the dependence of value u_m° on x°, which is determined by relationships (5.2) - (5.4), for the discharge of air into water at a high subsonic speed ($\rho^{\circ} \approx 650$) we finally obtain:

$$\frac{\rho_m u_m^2}{\rho_g u_g^2} = \frac{0.92}{x^{\circ} - 4.3} . \tag{5.6}$$

Comparison of the calculated curve of attenuation of the velocity head along the axis of the jet by formula (5.6), with the experimental data derived from reference /13/, indicates (Fig. 13) an acceptable accord between the calculation method developed here and the test, especially if the considerable scattering of experimental points as well as the approximation character of relationship (5.5) is considered. The dotted line in Figure 13 joins the curves of attenuation of the velocity head in the initial ($x^{o} \approx 2$) and the main ($x^{o} > 5.5$) area of the jet, and is traced conditionally.

Comparison of data on the change in the velocity head along the axis of a liquid gas jet with a corresponding curve for a submerged turbulent jet with constant density ($\rho^{o} = 1$), as in the case of a highly heated air jet, points out the invalidity of L. A. Vulie's /11/ assumption on the universality of the velocity head fields (impulse flux) in turbulent jets.

It should be noted that in the case of an isothermic jet propagating in the associated flow, the results of the calculation, based on the hypothesis of the universality of the ejection properties, with $m_o < 0.35$ are very close to calculated data obtained on the basis of G. N. Abramovich's /3/ theory. But these and other results are in good accord with experimental data. However, while the theory of G. N. Abramovich, due to the above indicated reasons, starts to differ from the test data with the approach of value m_o to one, the hypothesis on the universality of the ejection properties of the jet in this diapason of the change of parameter m_o also completely corresponds with the test.

In addition, calculated results by the proposed method with $m_o < 0.35$ are in satisfactory accord with the theory of jets developed by the author and published in monograph /3/. This refers to jets discharging from a nozzle at high speed, as well as to liquid gas jets. However, in this theory observation of anisothermal gas jets was performed with the assumption that the specific heat capacity c_p was constant, but that, at a considerable gas temperature ($\rho^{\circ} > 3$), it already did not correspond with actuality. If dependence of c_p on the temperature is considered in the calculation formulas as was done here, then the above indicated correspondence of calculations by the new method and the theory of gas jets /13/, where expansion of a jet was determined from relationship

$$\frac{db}{dx} \sim \frac{u_1 - u_2}{u_{cp}} \qquad \left(u_{cp} = \int_0^b \rho u dy \Big/ \int_0^b \rho dy\right)$$

will in all probability be observed, also, in cases of highly anisothermic gas jets.

If we consider that the hypothesis on the universality of the ejection properties satisfactorily corresponds with the results of experimental investigations of jets of essentially variable densities (plasma, liquid gas and supersonic jets), then the calculation method based on this hypothesis may be accepted as being sufficiently reliable.

The method proposed here may be used to solve analogous problems for plane-parallel gas jets, also. Furthermore, the calculation method based on the hypothesis of the universality of the ejection properties of turbulent gas jets appears to be also applicable to jets propagating into an area bound by solid walls (for example, a mixer chamber of the ejector) during analysis of gas jets developing in the drifting side flow, as well as during examination of problems of turbulent combustion in the flow. In conclusion the author takes this occasion to express his thanks to G. N. Abramovich for his very useful suggestions on this article, and his gratitude to E. Ya. Firsova for her role in the calculations.

Submitted 21 · VII 1960

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